Example 5: Find the domain and sketch the graph of $y = x^2 - 6x$.

Solution:

Given curve is,

$$y = x^2 - 6x$$

Since y is polynomial which is continuous on $(-\infty, \infty)$.

So,

Domain of $y = (-\infty, \infty)$.

Intercept: Put $y = 0$ then $x = 0$ and $x = 6$. And, put $x = 0$ then $y = 0$

Symmetricity: Clearly y is neither odd or nor odd.

Asymptotes:

Here,

$$\lim_{x\to a} y = \lim_{x\to a} x(x-6)$$

This shows $y\to\infty$ only when $x\to\infty$. So, y has no asymptotes.

Increasing/Decreasing:

Here:

$$y' = 2x - 6 = 2(x - 3)$$

For critical point, set

$$y' = 0 \Rightarrow x = 3.$$

Interval	$(-\infty, 3)$	$(3, \infty)$
Value of f '(x)	– ve value	+ ve value
Nature of curve of f(x)	Decreasing	Increasing

Here,

$$y(3) = 9 - 18 = -9$$

Concavity:

Here

$$y'' = 2$$

Clearly, $y'' \neq 0$. So, the curve y does not change its concavity behavior.

And, $y'' = 2 > 0$ for any x. So, the curve is concave upward

With these information, the sketch of graph of y is as: